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MATHEMATICAL MODELS OF DRYING PROCESSES IN A FLUIDIZED BED

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Mathematical models are described for drying processes in a fluidized bed with various idealizations. These may be applied to calculations on drying equipment involving a fluidized bed and to the development of an automatic process control system using computing techniques.

In developing mathematical models of drying processes in a fluidized bed with distributed parameters for a wide range of technical conditions, the problem arises of combining within the scope of a single model the kinetics of drying, i.e., the processes occurring in the individual particles, and the motion of the particles in the equipment.

This problem has been formulated for fluidized bed processes and solved in general form [1, 2] by introducing a multiphase space. The present paper is a further development in the same direction and deals with the application to a fluidized-bed drying process. The coordinates of the phase space may be any values characteristic of the process, their number being determined by the dimensions of the space. A sufficiently complete description of the drying process in a fluidized bed can be obtained in a five-dimensional phase space whose axes are particle moisture content, particle size, and the three space coordinates of the equipment.

This kind of description will allow for the finite rate of particle mixing, variation of grain size spectrum, drying kinetics, abrasion, entrainment, and precipitation of particles from the fluidized bed.

The unknown function for which the equations of the mathematical model are written is the particle probability density function $\rho(r, \omega, R, t)$, which, when multiplied by the particle density in the fluidized bed, is the number of particles with characteristic dimension R in unit R and with moisture content ω in unit ω, existing in unit volume at time t at the point in the equipment with radius vector r.

The equation for the function $\rho(r, \omega, R, t)$ is obtained from the material balance condition for the number of particles in an element of volume of the five-dimensional space under examination. We assume that particle mixing in the bed takes place according to the law of diffusion [3].

$$\frac{\partial \rho(\mathbf{r}, w, R, t)}{\partial t} = -\operatorname{div}[D \operatorname{grad} \rho(\mathbf{r}, w, R, t) - \alpha^* \rho(\mathbf{r}, w, R, t)] - \\ -\frac{\partial \rho(\mathbf{r}, w, R, t)}{\partial R}\frac{dR}{dt} - \frac{\partial \rho(\mathbf{r}, w, R, t)}{\partial w}\frac{dw}{dt} + q_s. \quad (1)$$

Equation (1) indicates that the rate of change of particle density in the volume element dr dω dR (density is understood in the generalized sense of the number of particles in unit volume of the five-dimensional space under examination) is determined by migration of particles from one section of the bed to another according to the law of diffusion (the first term of [1]), by a continuous transition process of particles from one size category to another due to attrition of the particle surface [the terms $(\partial\rho/\partial R)(dR/dt)$], and by continuous "migration" of particles along the imaginary moisture content (ω) axis because of drying [the term $(\partial\rho/d\omega)(d\omega/dt)$]. The coefficients D and a^* depend on the hydrodynamic conditions of motion in the equipment:

$$D = D(\Gamma), \ \alpha^* = \alpha^*(\Gamma). \quad (2)$$

The rate of reduction of size dR/dt may depend on the properties of the material and the particle surface, on the moisture content, and on other factors.

Introducing the simplest attrition mechanism—rate of change of particle volume proportional to its surface—we arrive at the following relation:

$$dR/dt = -k_{sr}, \quad (3)$$

where k_{sr} is a coefficient independent of particle size.

Next the rate of change of moisture content, or rate of drying, is determined. Equation (1) must be satisfied by the initial and boundary conditions, the actual form of which depends upon the process itself and the construction of the equipment.

Let us examine some idealizations which may be used in applying (1) to actual drying processes in a fluidized bed, confining ourselves to models in which the kinetics of drying are reflected. We thus eliminate from consideration the zero-dimension model, into which there enters only the rate of drying averaged over the whole residence time of the particles in the bed.

1. The process is homogeneous throughout the equipment volume. All the particles are identical in size. Particle size reduction is ignored.

In this case (1) becomes

$$\frac{\partial \rho(w, t)}{\partial t} = -\frac{\partial \rho(w, t)}{\partial w} \frac{dw}{dt} + \Phi_i(w, t) - k_d \rho(w, t), \tag{4}$$

where

$$k_d = \int \Phi_i(w, t)\, dw / \int \rho(w, t)\, dw. \tag{5}$$

Let us determine the rate of drying dw/dt for two cases.

(1) Drying proceeds at a constant rate:

$$dw/dt = -N. \tag{6}$$

Under the assumption that heating of particles arriving in the bed is instantaneous and that the bed height is such that the heat carrier leaves it at a temperature equal to that of the material, i.e., the wet bulb temperature, we obtain the following expression for N:

$$N = \frac{G_d c_d (T_0 - T_{wb}) - Q}{r_v a \int \rho(w, t)\, dw}. \tag{7}$$

(2) Drying is periodic, proceeding first at a constant and then at a changing rate.

Expression (7) for the rate in the first period ceases to be valid, since now the temperature of the heat carrier at the bed outlet is not equal to the wet bulb temperature, but exceeds it, because the average temperature of the particles increases because of the particles dried in the second period; the heat is not divided equally among all the particles, which differ from one another in temperature.

The drying rate in the second period must be higher than that of (7), since the average temperature of the heat carrier over the height of the bed increases, and therefore the average temperature drop between the carrier and the particles dried in the second period and having temperatures equal to the wet bulb temperature increases; hence, there is also an increase in the drying rate, which is determined by the amount of heat which a particle receives in unit time.

The expression for the average temperature of the carrier as a function of the height of the fluidized bed under conditions of full utilization of heat for various average particle temperature $\overline{T}_p$ has the following form [4]:

$$\overline{T}(H) = (T_0 - \overline{T}_p) \frac{G_m c_m}{\alpha_T S \rho H_b} \left[1 - \exp\left(-\frac{\alpha_T S \rho H_b}{G_m c_m}\right)\right] + \overline{T}_p. \tag{8}$$

Then the drying rate in the first period is determined by

$$N = \alpha_T S [\overline{T}(H) - T_{wb}] / r_v a. \tag{9}$$

To determine the drying rate in the second period, we make use of the approximate equation derived in [5]:

$$dw/dt = -N\psi; \ w < w_{cr} \tag{10}$$

Here N is determined from (9), and ψ – the reduced drying rate – is given by

$$\psi = (w - w_e)^m / [A + \beta (w - w_e)^m], \tag{11}$$

where m, A, β are dimensionless coefficients independent of the moisture content of the material.

The average temperature of the particles in the bed $\overline{T}_p$ in (8) may be determined from a knowledge of the particle temperature distribution $\rho(T)$, which is connected with the particle moisture content distribution $\rho(\omega)$, since there is a single-valued relation between the temperature and the moisture content of the material $T(\omega)$ during drying [5]:

$$\rho(T) = \rho(w)\, (dT/dw)^{-1}. \tag{12}$$

Then

$$\bar{T}_p = \frac{\int (c_{mat} + c'_w w) T(w) \rho(w) dw}{\int (c_{mat} + c'_w w) \rho(w) dw}. \quad (13)$$

In order to compute the heat going into heating the material, which is assumed to be instantaneous, we take the initial temperature of the heat carrier to be equal to some effective temperature T_{0eff}:

$$T_{0\,eff} = T_0 - Q/G_m c_m \quad (14)$$

Thus, depending on the drying conditions, the drying rate in (4), $d\omega/dt$, will be determined either by (7) or by (9) and (10), together with (8), (11) and (13).

It is convenient to investigate (4) on an analog computer. In setting up the equation only one independent variable is possible, namely, time. Otherwise the equation must be represented in finite differences. Figure 1 shows a block diagram of the mathematical model according to (4), expressed in finite differences. The diagrams for drying rate N and reduced drying rate ψ are not shown in Fig. 1 nor in the later diagrams (Figs. 2-4); their specific form must be made to correspond to the conditions of the actual process.

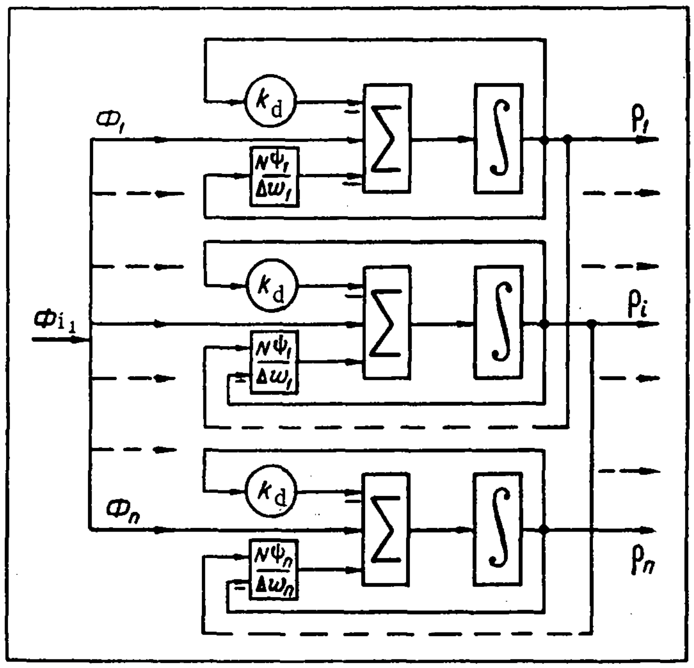

Fig. 1. Block diagram of a mathematical model for the drying process in a fluidized bed taking into account the kinetics of drying.

2. The process is homogeneous throughout the equipment volume. The particles have a size distribution. Attrition is taken into account.

In this case (1) becomes

$$\frac{\partial \rho(w, R, t)}{\partial t} = -\frac{\partial \rho(w, R, t)}{\partial w}\frac{dw}{dt} - \frac{\partial \rho(w, R, t)}{\partial R}\frac{dR}{dt} + \\ + \Phi_i(w, R, t) - k_d \rho(w, R, t). \quad (15)$$

The rate of attrition dR/dt is given by (3).

Under the assumption that the heat is divided between particles of different sizes in proportion to their surface areas in the bed, the drying rate, when for all the particles in the bed drying occurs only in the first period, is given by

$$\frac{dw}{dt} = -\frac{G_m c_m (T_0 - T_{wb}) - Q}{a(R) r_v} \frac{R^2}{\iint R^2 \rho(w, R) dR dw}. \quad (16)$$

If drying occurs in the first and second periods, the drying rate when $\omega \geq \omega_{cr}$ is determined by (9), where $S = S(R)$, $a = a(R)$, and when $\omega < \omega_{cr}$, by (10).

We must put $\int S(R)\rho\, dR$ instead of $S\rho$ in expression (8) for $\overline{T}(H)$ and carry out an additional averaging with respect to R in the formula for $\overline{T}_p$.

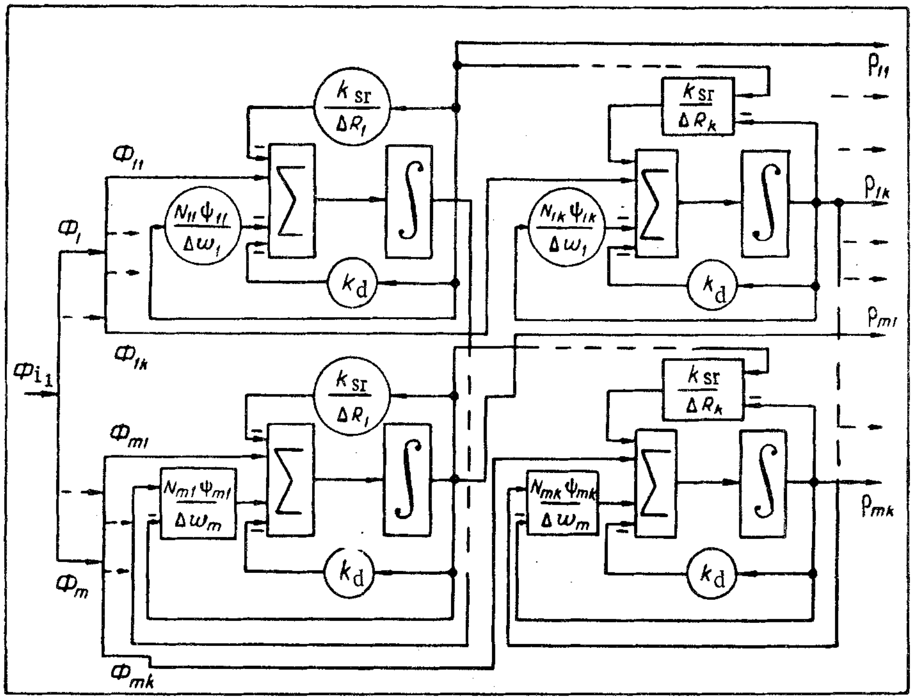

Fig. 2. Block diagram of a mathematical model for the drying process in a fluidized bed taking into account attrition and the kinetics of drying.

The block diagram of the mathematical model according to (15), represented in finite difference form, is shown in Fig. 2.

3. Allowance is made for the finite rate of migration of particles along one of the horizontal coordinates of the equipment–the length. The particles have a size distribution. Attrition is taken into account and also solid carry-over.

Equation (1) becomes

$$\frac{\partial\rho(x, w, R, t)}{\partial t} = D\frac{\partial^2\rho(x, w, R, t)}{\partial x^2} - \frac{\partial\rho(x, w, R, t)}{\partial w}\frac{dw}{dt} - \frac{\partial\rho(x, w, R, t)}{\partial R}\frac{dR}{dt} - \alpha\rho(x, w, R, t). \tag{17}$$

We determine the attrition rate from (3).

To calculate the heat going into heating the material, we introduce a heating zone, the length of which is given by the following expression:

$$x' = \frac{\int(c_{mat} + c'_w w)(T_{wb} - T_{m,0})\,G\rho(w)\,dw}{G_m c_m (T_0 - T_{wb})}. \tag{18}$$

Inside the heating zone $dw/dt = 0$.

Outside the heating zone the drying rate will be determined either by (16) (when $Q = 0$), if the number of particles with subcritical moisture content is negligibly small in the section of the bed in question, or by (9) and (10), if the number becomes appreciable.

Figure 3 shows a block diagram of a mathematical model in accordance with (17), represented in finite differences.

In constructing the diagram it was assumed that $dR/dt = -k_{sr} = 0$. If particle attrition is important in the drying process, the diagram can easily be supplemented by the missing coupling in a similar way to Fig. 2. The input particle spectrum is divided into p fractions according to size. The diagram consists of p parallel connected channels. Figure 3 shows the structure of one channel, the k-th. The structure of the remainder is similar.

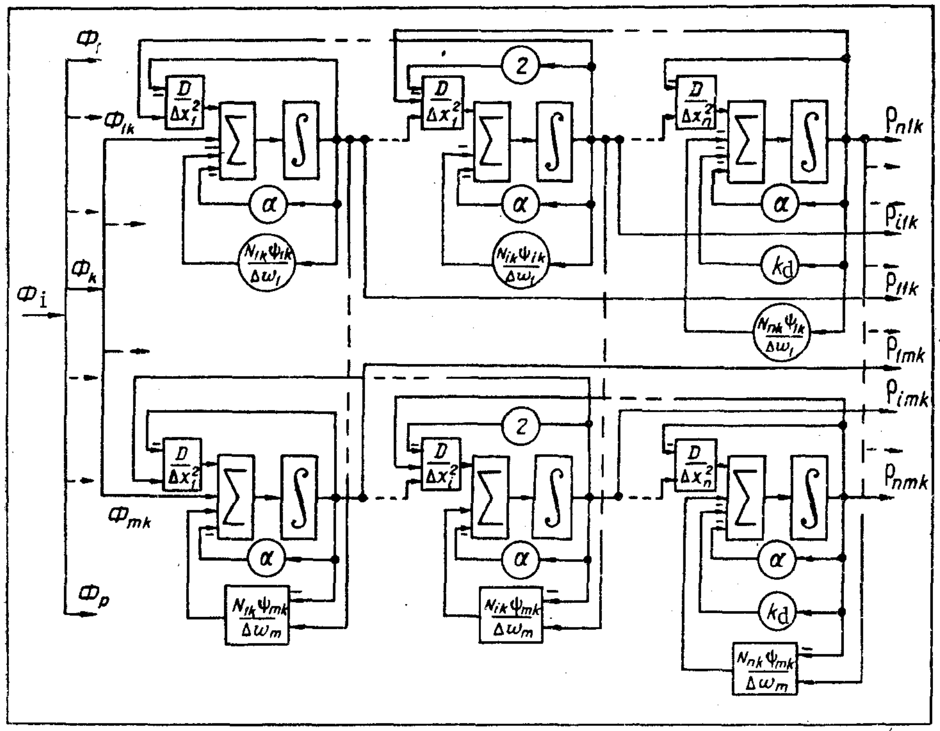

Fig. 3. Block diagram of a mathematical model for the drying process in a fluidized bed taking into account the kinetics of drying and the distribution with respect to a space coordinate of the equipment for a polydisperse material.

4. In the three foregoing cases we examined the problem of locating particles according to moisture content and any of the other coordinates (particle size—case 2, particle size and equipment coordinate—case 3).

Often the interest is not in the distribution of particles according to moisture content, but only in the average particle moisture content in the equipment or in different sections of it, over all the particles or within separate fractions. We may then construct the mathematical model for the drying process in such a way as to obtain the desired average moisture content immediately on solving it, whereas in the above cases additional averaging ($w = \int w \rho \, dw / \int \rho \, dw$) was necessary to obtain this value.

In constructing a mathematical model with averaging over moisture content we take into account the finite rate of migration of particles along one of the horizontal coordinates of the equipment (the process is assumed to be homogeneous along the other two coordinates) and solid carry-over from the fluidized bed. The input particles have a size distribution. Attrition is neglected.

With the above assumptions, in the case of a continuous particle size spectrum the mathematical model for the drying process will consist of an infinite number of pairs of equations:

$$\frac{\partial \rho(x, R, t)}{\partial t} = D \frac{\partial^2 \rho(x, R, t)}{\partial x^2} - \alpha \rho(x, R, t); \tag{19}$$

$$\frac{\partial [w(x, R, t)\rho(x, R, t)]}{\partial t} = D \frac{\partial^2 [w(x, R, t)\rho(x, R, t)]}{\partial x^2} + q_s - \alpha w(x, R, t)\rho(x, R, t). \tag{20}$$

In practice, we always consider the particle size spectrum of the process material to be discrete. Then the mathematical model will consist of 2p equations analogous to (19) and (20) (p is the number of fractions into which the material is divided with respect to size).

The unknown functions, with respect to which the equations of the mathematical model are solved, are the density distribution for particles of various sizes along the length of the bed $\rho(x, R, t)$, and the average moisture content of particles of various sizes in various sections of the bed and at the outlet of the equipment $w(x, R, t)$.

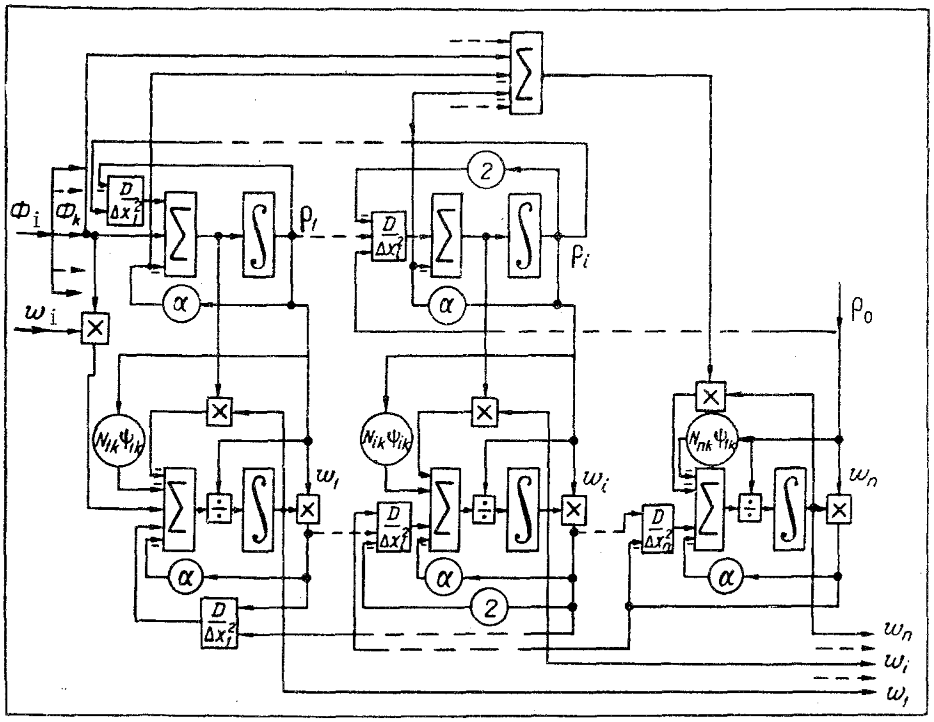

Fig. 4. Block diagram of a mathematical model for the drying process in a fluidized bed taking into account the distribution of average particle moisture content along an equipment coordinate for the polydisperse case.

The density of the sources q_s is connected with the drying rate, determined in the foregoing case (Paragraph 3), by the relation

$$q_s = \frac{dw}{dt} \rho(x, R, t). \tag{21}$$

The value of dw/dt from (16) must be inserted into (21), if the average moisture content of the particles of any at section x is above critical. When this condition does not hold, dw/dt must be determined according to (9) for $w \geq w_{cr}$ and (10) for $w < w_{cr}$.

Figure 4 shows the block diagram of a mathematical model according to (19) and (20), expressed in finite differences. Particles of the k-th fraction are shown in this figure. The drying of particles of other sizes follows a similar scheme.

NOTATION

R—characteristic dimension of particle; ω—moisture content of particles; $r = \{x, y, z\}$—radius vector of particle; D—mixing factor; a^*—directed velocity of fines in fluidized bed; Γ—combination of values determining hydrodynamics of fluidized bed; k_d—discharge coefficient; Φ_i—particle flux at inlet; Q—amount of heat expended in heating material to wet bulb temperature; T_{wb}—wet bulb temperature; G_m, c_m—mass flow rate and specific heat of heat carrier; r_v—heat of vaporization; ω_{cr}, ω_e—critical and equilibrium moisture content; G_p—mass flow rate of solid (dry mass); α—carry-over factor; T_0, $T_{m,0}$—initial temperature of heat carrier and material; c_{mat}, c'_w—specific heat dry material and of water; k_{sr}—size reduction coefficient; α_T—heat transfer coefficient; a, S—particle mass and surface area; ψ—reduced drying rate; $\overline{T}(H)$—average temperature of heat carrier over bed height; $\overline{T}_p$—average temperature of particles in bed; m, A, β—dimensionless coefficients; x'—length of heating zone.

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